# Structural Optimization and Free form Design

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## Summary

Free form design is an important trend in contemporary architecture which aims to overcome the geometric limits of traditional architecture. With the advent of free form design topics like structural efficiency and reliability play a central role in the design process. The adoption of uncommon geometry (often extended in 3D space) make difficult the adoption of classical methodologies for structural analysis. New techiques based on optimization methods can bring a useful contribution in both the architectonic and structural design. In this paper an optimization method based on genetic algorithm is presented and several practical applications are shown and discussed. The complexity due to the large amount of calculations needed by this kind of approach have been exceeded by the adoption of parallel computing strategies that seems to be powerful, scalable with the size of the problem and also valuable from the economic point of view [1].

The present technique can be successfully adopted for sizing and topology optimization of truss structures in real size.

**Keywords:** structural optimization; genetic algorithms.

## 1. Introduction

Over the years a large number of optimization techniques has been introduced. Historically the importance of structural optimization has been first recognized by the aerospace industry that is particularly interested in the reduction of structural weights.

Nowadays the interest is still high because of the growing demonad for lightweight, efficent and low cost structures.

In general terms a generic optimization problem can be expressed as a minimization of an objective function $f(x)$ subject to constraints in its own variables $x$:

$min\, f(x)$, $\forall x \in \Re^n$ subject to $c_i(x) = 0$, $c_j(x) \geq 0$

with $i \in \Upsilon$, $j \in \Xi$ where $c_i$ and $c_j$ are functions in $x$ and represent the constraints [2].

In structural optimization the variables are often related to stresses, displacements, vibration frequencies or others. From a structural point of view, many authors agree on the following classification about different levels of optimization [3]:

- **size** optimization deals with minimization of one ore more response variable (tension, deformation or others) acting on one ore more design variables (such as thickness for a plate or cross section of a bar) while respecting some conditions (equilibrium, restraints and so on);
- **shape** optimization aims to find the optimal shape of a domain which is a design variable;
- **topology** optimization for continuum structures deals with the number, position, shape of holes and topoology of the domain.

Commonly used objective functions are weight, displacements or stresses.

Many techniques have been introduced in order to solve the problem expressed in the form (1.1). Among them the so-called search methods are numerical search techniques that start from an initial design and proceed in small step to improve the value of the objective function in or the degree of compliance with the constraints, or both. The search is terminated when no progress can be made in improving the value of the objective function or the degree of compliance with the constraints or both. Other methods adopt the necessary conditions that must be satisfied at minimum but many others are available [4].

In the past *Optimality Criteria* (OC) and *Mathematical Programming* (MP) methods gained a lot of popularity among researchers. Unfortunately neither of them is so robust and efficient to be applied in general (or almost in a large class of problems). They are mathematical strategies that encounter intrinsic difficulties in structural problems and this is mainly due to the fact that they need to compute derivatives of functions which are, very often, not regular. Moreover this kind of approach can't take into account the uncertainty and approximations that are commonly present in real applications.

Many other tenchniques can be adopted, including the *Evolutionary Structural Optimization* (ESO) or the *Homogenization Method* (HM).

In the opinion of the authors of the present work one of the most promising approaches seem those based on evolutive algorithms that take their inspiration from the observation of natural events: *Genetic Algorithms* (GA) and *Genetic Programming* (GP) represent a totally different approach if compared with the mathematical techniques cited above.

# 2. Genetic algorithms

*Genetic Algorithms* are based on the natural law of the survival of the fittest (Darwin's theory). This methodology create a population of randomly generated individuals. The individuals reproduce according to the laws of nature and the population evolve from generation to generation. At the end of the process, the individuals with highest fitness represent the optimal solution.

## 2.1 Objective Function

Several objective functions can be considered. The most common in the field of structural optimization are:

- minimization of weight;
- maximization of stiffness;
- some control on free vibrations;

The previous items can also be combined leading to a multiobjective optimization.

## 2.2 The solution procedure

The informations belonging to each individual are encoded in chromosomes. The data are combined together during the reproduction phases. The most important mechanisms that allow exchange of chromosomal informations are the important operators:

- crossover;
- mutation;
- inversion.

### 2.2.1 Example

In its basic form to make this approach valuable it is necessary to represent the possible combinations of the variables in terms of bit strings and to found an objective function to evaluate the fitness of each solution. As an example, for a simple problem with only 4 design variables that can be expressed in the form

$$\textit{minimize } f(\mathbf{x}),\ \mathbf{x} = \{\ x_1, x_2, x_3, x_4\ \},$$

and a possible binary string representation could be:

$$\underbrace{0\,1\,1\,0}_{x_1}\qquad \underbrace{1\,0\,1}_{x_2}\quad \underbrace{1\,1}_{x_3}\qquad \underbrace{1\,0\,1\,1}_{x_4}$$

In general terms a continous variable $x_i \in (x_i^U, x_i^L)$ to be approximated (with an accuracy between two adjacent values $x_{incr}$) requires $m$ binary digits such that

$$2^m \geq 1 + (x_i^U, x_i^L)$$

where $m$ can be taken as the lowest integer that satisfies the expression above.

The advantage of the GAs is that they present a lower risk to getting stuck at local minimum and they are able to give not a single solution but a set of optimal designs. The main procedure, at outer level, can be summed up as follow:

1. The size of population is chosen and the values of variables are assigned with randomly chose values for the bits.
2. The individuals with best values of objective functions are chosen for *reproduction*.
3. The new generation is created applying *crossover*: bit informations are exchanged between the indivduals found at the previous step.
4. From time to time, random alteration of string are performed (*mutation*).
5. The objective function is evalutated for all the individuals of the new generation and iterations are performed until there are no more improvements.

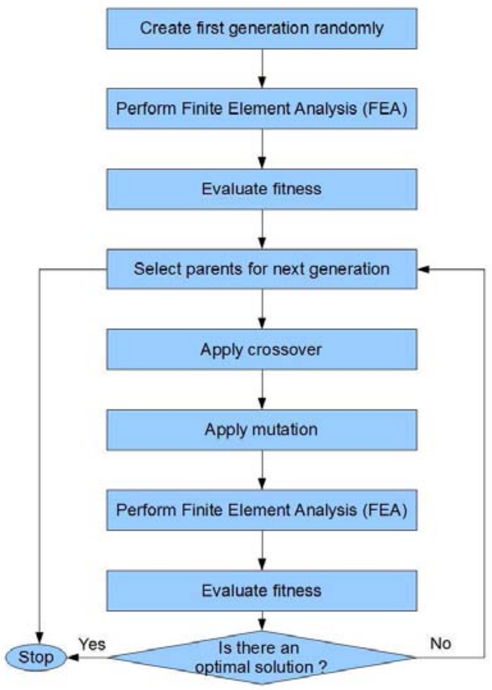



Figure 1: Optimization Procedure

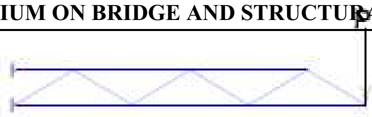
A flow chart of the procedure can be seen in Figure 1. Typically the objective function is a fairly simple function such as the computation of the total weight of the structure (expressed in function of the design variables). Moreover the design is often subjected to a certain number of constraints (related to stresses, displacements, buckling loads, vibration frequencies, etc). The relations between constraints and design variables can be very complex and it is necessary to adopt a finite element analysis in order to evaluate them.

## 3. Numerical simulations

To evaluate the efficiency of the proposal approach several numerical simulations has been performed in both 2D and 3D truss structures. In the following some results are presented.

### 3.1 2D truss cantilever

#### 3.1.1 Description of the problem

This example deals with both single and multiobjective structural optimizations. The procedures has been applied to a real 2D steel str  ers and loaded by a nodal force $P$.

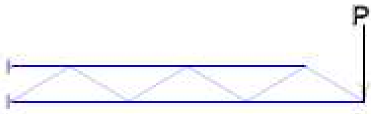



Figure 2: 2D truss cantilever

Three different objectives have been considered:

- minimization of total weight;
- minimization of vertical displacement of the directly loaded node;
- a combination of both of previous objectives.

The design variables are the cross area sections. The members are supposed to have a circular cross section, so two are the parameters to evaluate:

- the external diameter $D$;
- the thickness $t_i$;
- resulting in 24 design variables for each solution. The constraints are:
- the member resistance;
- $0.1\ m < D < 0.7\ m$;
- $0.002\ m < t < 0.05\ m$;

For every member the resistance has been computed according to allowable stress method ( $\sigma \leq \sigma_{allowable}$), taking into account the resistance penalization for $t > 40$ mm. The instability problem has been neglected. Both the external load $P = 100$ kN and the varying self weight of the structure are considered by the optimization procedure as well.

We've considered S275 grade steel with the following data:

- $E = 206000\ MPa$;
- Shear Modulus G = 80000 $MPa$;
- Weight for unit of volume $\gamma = 78.50\ kN/m^3$;
- Poisson's coefficent $\nu = 0.3$;
- $\sigma_{allowable} = 190\ MPa$ ( $t < 40\ mm$ );
- $\sigma_{allowable} = 170\ MPa$ ( $t \geq 40\ mm$ );

The structural geometry can be seen in the following figure.

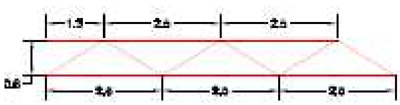


Figure 3: structural geometry

The adopted parameters for genetic algorithm are:

- Population size $s = 100$
- Selection algorithm: *roulette wheel*;
- Selection probability $p = 0.05$;

- Crossover probability $p = 0.5$;
- Mutation probability $p = 0.1$;

### 3.1.2 The optimization process

The structure has been modeled with twelve truss elements with the following numbering scheme:

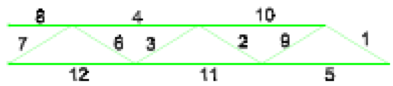



Figure 4: numbering scheme

**Objective 1: weight optimization**

The objective is the reduction of the total weight of the structure. The best result is 8368 N, the worst is 143009 N. The Figure 5 show the history of the objective function (including both feasible and infeasible solutions). It can be seen that the solution seem to behave randomly in the initial population and improve (on the average) going further the following generations. The optimal result is described in Table 1.

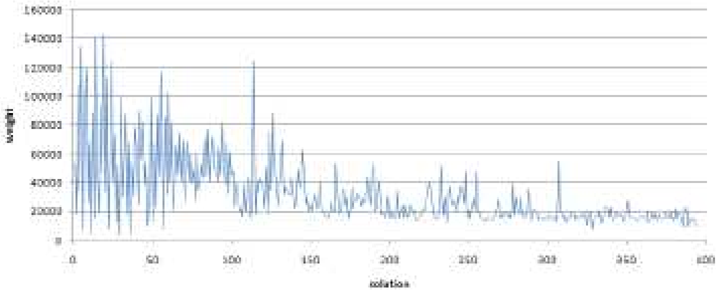


Figure 5: weight history

**Objective 2: displacements optimization**

The same structure has been studied in order to minimize the vertical displacement of node no. 2. The best result is detailed in Table 2. The corresponding displacement is 0,003894162 m. The following Figure shows the history of the objective function (including both feasible and infeasible solutions). Usually the peaks in the value of the displacements represent solutions belonging to the infeasible domain.

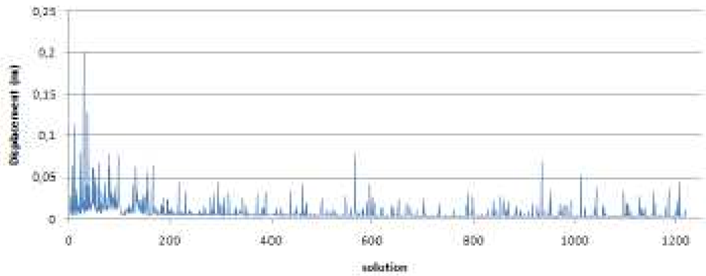



Figure 6: displacements' history

**Objective 3: multiobjective optimization**

- minimization of total weight;
- minimization of vertical displacement of the directly loaded node.

In order to understand what happen during the optimization process we will focus our attention in member number 8. This is only for the sake of simplicity. The same consideration could then be extended also to the other members. Moreover, since the multi objective problem come from only two different objectives it is possible to use a plane graph load-displacement. This makes easier to understand graphically the performance of the solutions. The following figure shows the position of the best solution in the graph load-displacement. From the numerical result it can be seen that the weight varies in the range 10516,45÷117658,5 N and the displacements remain inside the interval 0,00460÷0,043036 m. Figure 7 shows the points of the research space that have been investigated: each point represent a solution of the design space

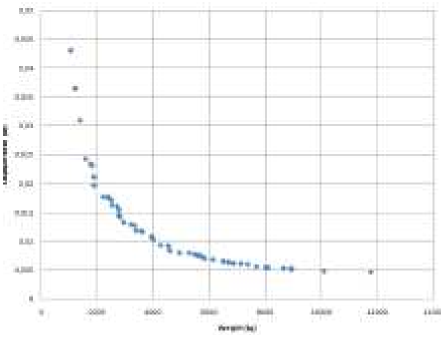


Figure 7: Weight-displacement for optimal solutions

*Table 1: Weight optimization*

| Element | Diameter (m) | Thickness (m) |
|---|---|---|
| 1 | 0,654 | 0,004 |
| 2 | 0,541 | 0,004 |
| 3 | 0,560 | 0,004 |
| 4 | 0,391 | 0,004 |
| 5 | 0,278 | 0,004 |
| 6 | 0,146 | 0,004 |
| 7 | 0,109 | 0,004 |
| 8 | 0,597 | 0,004 |
| 9 | 0,522 | 0,004 |
| 10 | 0,428 | 0,004 |
| 11 | 0,315 | 0,004 |
| 12 | 0,240 | 0,004 |

*Table 2: Displacements optimization*

| Element | Diameter (m) | Thickness (m) |
|---|---|---|
| 1 | 0,248 | 0,045 |
| 2 | 0,382 | 0,050 |
| 3 | 0,671 | 0,049 |
| 4 | 0,700 | 0,050 |
| 5 | 0,337 | 0,037 |
| 6 | 0,639 | 0,047 |
| 7 | 0,658 | 0,049 |
| 8 | 0,685 | 0,050 |
| 9 | 0,646 | 0,037 |
| 10 | 0,517 | 0,044 |
| 11 | 0,700 | 0,045 |
| 12 | 0,700 | 0,050 |

## 3.2 Double hinged 2D truss

### 3.2.1 Description of the problem

This example deals with the structure represented in Figure 8 that has been discretized with 29 2D truss elements.

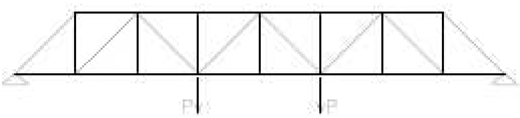


Figure 8: Double hinged 2D truss

### 3.2.2 Data

The data for the material are:

- Steel type: S235
- Modulus of elasticity $E = 206000\ MPa$
- Shear modulus $G = 80000\ MPa$
- Weight for unit of volume $\gamma = 78.50\ kN/m^3$
- Poisson's coefficent $\upsilon = 0.3$
- $\sigma_{allowable} = 160\ MPa$ ( $t < 40$ mm );
- $\sigma_{allowable} = 140\ MPa$ ( $t \geqslant 40$ mm );

The load P is 100000 N.

The parameters for the genetic algorithms are:

- Population size $s = 100$
- Maximum number of generations = 2000;
- Selection algorithm: *roulette wheel*;
- Elitism activated;
- Mutation probability $p = 0.1$;

The algorithm evaluates the self weight of the structure automatically and takes ino account the instability of compress members according to CNR-UNI 10011 national code. Several kinds of optimizations have been performed.

**Sizing Optimization**

The cross sections are supposed to be hollow circular sections. They are defined by two parameters: the diameter $d_i$ and the thickness $t_i$ with $i = 1, 2, \dots 15$ (for simmetry reasons, only 15 members have been considered). The thickness is assumed to be constant and equal to 0.005 *m*, while the diameter must satisfy the constraint $0.1\ m \leqslant d_i \leqslant 0.2\ m$. The numerical simulations give as the optimimum result a weight of 8547.09 *N* corresponding to a weight reduction (respect to the first random generation) of 20.6% (see Figure 9).

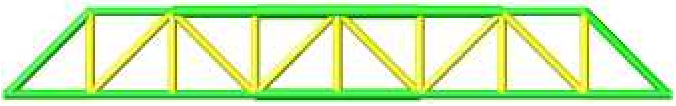


Figure 9: Sizing optimization

**Topology Optimization**

In this optimization problem the structural nodes are allowed to move vertically and horizontally from their original position by a quantity $\Delta \pm 0.8\ m$. All the members share the same cross section which is fixed (hollow circular section with diameter $d = 0.1143\ m$ and the thickness $t = 0.005\ m$) and remains constant. The initial geometry is shown in Figure 10.

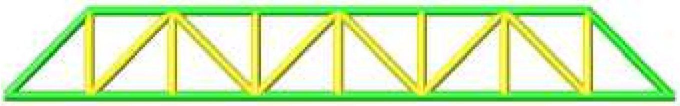


Figure 10: Topology optimization: initial solution

The numerical procedure gives as the *optimum* result a weight of 10121.3 N. In such a case the weight reduction (compared to the first randomly created generation) equals to 6.6%. The final result can be seen in Figure 11.

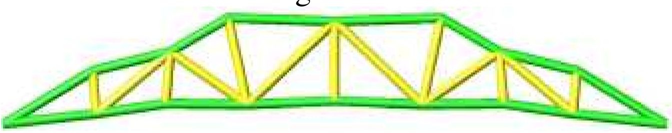


Figure 11: Topology optimization. Final result

### Combined Optimization

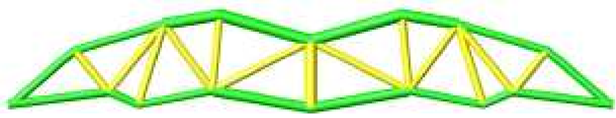


Figure 12: Combined optimization. Final result.

This application is a combination of the two previous optimizations. Here the cross sections are supposed to be hollow circular sections. They are defined by two parameters: the diameter di and the thickness $t_i$ with $i$ = 1, 2, ...15. The thickness is assumed to be constant and equal to 0.005 m, while the diameter (in meters) must satisfy the constraint $0.1 \leqslant d_i \leqslant 0.2$. At the same time the structural nodes are allowed to move vertically and horizontally from their original position by a quantity Δ = ± 0.8 m. The optimum result can be seen in Figure 13 and corresponds to a weight of 8728.69 N. The weight reduction is about 23.1% of initial value. The whole optimization process can be seen in Figure 14. It is shown that most of the weight reduction is performed during the first 100 iterations.

### Comparison

A comparison among sizing, topology and combined (sizing+topology) optimization can be seen in the following Figure. For this problem, after 2000 generations. The sizing optimization seems to behave better thatn topology optimization, but in terms of relative weight reduction both of them are outperformed by the combined technique.

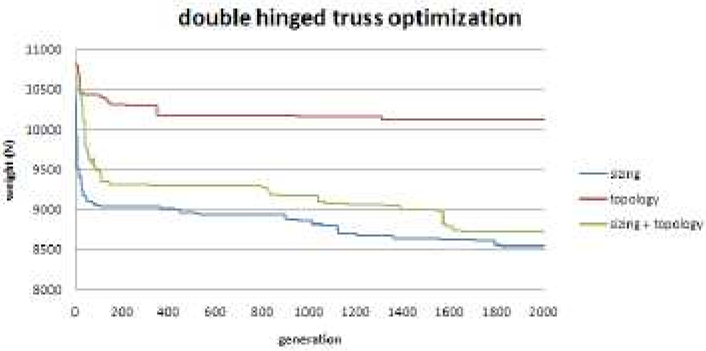



Figure 13: Final comparison

## 4. Conclusions

An approach for structural optimization based on genetic algorithms has been presented. Several applications have been described and a comparison on different techniques has been shown. The proposed approach appear to be valuable for structural optimization of 3D structures in real size [1] and could be a valuable support for modern architectural free-form design, being able to relate architectural geometry with structural performances.

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