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TITLE : 'ON THE DEFORMATIVE STATE OF PNEUMATIC STRUCTURES'

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SUMMARY :

In this paper a new calculation method of the deformative sta
te of pneumatic structures is suggested. At a first stage the
method is essentially based on the kinematic schematization
of the movements; at a second stage it takes into account the
effect of the material elasticity, thus solving the problem
of 'big displacements', through a simple iterative method. Spa
cial pneumatic structures and sufficiently long pneumatic
structures, which can be reduced to a plane problem, are con-
sidered.

1.- INTRODUCTION

In projecti g pneumatic structures, besides the basic problem of the definition of the shape [1] , which is typical of all membrane roofings, it's extremely important to determine the deformations due to accidental factors, such as wind and snow.

Considering that the functioning of this type of buildings re quires a continuous supply of electrical power, it's necessa ry to determine the relations between the internal pressure of the membrane and the characteristic or actual intensity of accidental loads, in order to control the shell movements. This basically for two reasons :

1) To guarantee a sufficient safety margin before reaching the limit utilization state (exoessive deformation) [2] , which sometimes coincides with the last state (just consider the pos sible material lacerations if it gets in touch with objects in side the shell).

2) To provide a correct and consequently economical function- ing of the construction, by keeping the internal pressure as low as possible with reference to the actual values of live loads.

The effect of external loads on the resisting system, makes it necessary to define, even if in a deterministic way, the di- stribution and intensity of external factors, as well as to provide a quick and easy calculation instrument which, given the loads, enables to find out the deformed configuration of the structure, under conditions of great deformations.

As far as the first aspect is concerned, it is observed that the snow load can approximately be calculated on the basis of average intensities and plausible distribution, whereas simi- lar criteria for the wind are much more difficult to find.

Infact in order to determine the distribution of the external pressure due to an air current hitting the structure , very complex problems concerning the dynamic behaviour of structu- res, are to be faced.

Yet recent experiences [3] on the dynamic reactions of pneuma- tic structures, carried out in wind tunnels on models with a long cylindrical shape (ratio between maximum and minimum di- mension in the plan equal to 2) seem to indicate the possibi- lity of relatively simple schematization.

Infact, as far as movements are concerned, it was observed that dynamic effects have little importance if compared with the values determined through quasi-static calculations, based on the peak speed of the wind, especially at high inflation pressures. Moreover, again with regards to movements, theoretical anticipations based on a field of external pressures measured on a rigid model, gave results which were satisfactorily in accordance with the experimental indications. This was true also for situations that simulated the wind speed of 30 mt/sec and the (weak) internal pressure of 18 mm of water.

On the basis of this experimental data, the quasi-static calculations referred to a distribution of average pressure coefficients, result reliable.

Although this problem is still matter of research, we shall make reference to the overmentioned indications in order to study some problems of research of the deformative state.

2.- DETERMINATION OF THE DEFORMATIVE STATE IN THE PLANE CASE.

2.1 Some observations on the various evaluation method

In many cases pneumatic structures have long plans, so that the central section can be considered part of a membrane of indefinite cylindrical shape, not being seriously affected by the end sections, which are usually closed by nail easements.

It is therefore correct to takle these cases on the basis of the study of plane structures.

Many contributions were made to the solution of this problem especially as far as the effect of wind is concerned.

M.Uemura [4] studied cylindrical structures with circular directrix, without bending stiffness, under the action of the wind. He didn't consider the proper weight of the membrane in relation with the internal pressure, but took into account the elastic deformability of the material.
Assuming that the external pressure $p(\theta)$ (θ angle at center) is regularly distributed on the undeformed surface according to a law, such as $p(\theta) = C(\theta) \frac{1}{2} pv^2$, considering that the material stress S, according to the hypothesis, doesn't vary on the membrane, he finds out the deformed configurations, by developping the load in Fourier series (up to the 15th term), and by adopting a solution criterion of energetic type.

In that work, always assuming the function C (Θ) referred to
the undeformed state as unchanged, several cases are studied
which point up the influence of the Reynolds number.

Always in the framework of plane problems, iterative methods
are particularly interesting R.D.Parbery [5] faces the pro-
blem with reference to cylindrical membranes through the nu-
merical solution of differential equations (fig.1) :

$$\frac{d\varphi}{dl_{ind}} = -\frac{f(\sigma)}{S_. + p, z}\,(p - p, \cos\varphi)$$

$$\frac{dx}{dl_{ind}} = f(\sigma)\cos\varphi$$

$$\frac{dz}{dl_{ind}} = f(\sigma)\,\text{sen}\,\varphi$$

Here apart from the obvious meaning of some symbols, indicates
the ratio between the length of the deformed infinitesimal seg
ment (def) and the initial length (ind), i.e.
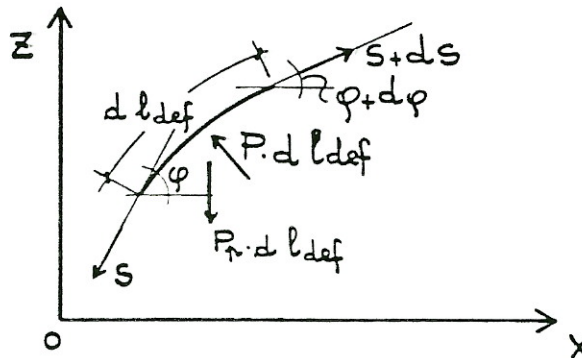
$$f(\sigma) = d\,l_{def}/d\,l_{ind}$$

Fig. 1

The iterative numerical solution obtained through the method
Runge-Kutta, and later developed through the method Newton-
Raphson, enables to find out the values φ (0) and S (0) at
the origin, which, provided that the conditions at the ini-
tial boundary are respected, also satisfy the conditions at
the opposite boundary condition.

In this work an interesting comparison is carried out by means
of a discrete iterative procedure suggested by B.H. Harrison
[6], following the same technique of the research of suita-
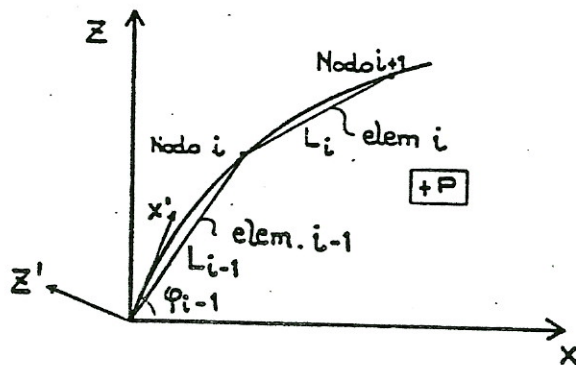
ble values at the origin (fig.2).

Fig. 2

Particularly in the case of unextensible membranes, the dis-
crete method corresponds to un approximate method of the se-
cond order, as in the equilibrium equation of the joint i ap
pear the effects of the loads affecting both the elements
i-1 and the element i; these are however assumed as an expres
sion of the angle φ_i, which in a first approximation, is ob-
tained as a function of the mechanical and geometrical para-
meters of the element i.
V.V. Yermolov [7] too gives the iterative solution of an un-
extensible pneumatic structure with negligeable weight under
the wind action, also graphically.

Considering the membrane discrete, having ascertained that
the stress doesn't vary along the edge, and that the force F
acting on the joint i shall always, for the equilibrium, co-
incide with the bisectrix of the angle formed by the elements
i-1 and i, Yermolov still through the iterative method, deter
mined the values of initial parameters satisfying the edge
conditions at the other extremum.

Yet this method is not so accurate, because the intensity of
the force P acting on the joint i is referred to the undefor
med edge.

P. Spinelli [9] suggests a substantial development of this me
thod; having also recourse to a display graphic, after analy-
sing more general load conditions, he solves the problem, par
ticularly as far as wind load is concerned, by continuously ad
justing the external factors both in direction and intensity.

2.2 The suggested solution method

Still within the framework of discrete solutions, the solution of the problem can be given by iteratively solving the equation system which expresses the point equilibrium conditions, the movements of the joints being assumed as unknown values.

In fact, considering the membrane deformability, and neglecting sufficiently small differences, we can write the following equilibrium equations for the generic joint i of the system :

(a)
$$
\begin{cases}
\dfrac{S_{i-1}}{L_{i-1}^{\cdot} + \Delta L_{i-1}}\, \Delta x_{i,\,i-1} + \dfrac{S_i}{L_i^{\cdot} + \Delta L_i}\, \Delta x_{i,\,i+1} = P_{x,i} \\[2ex]
\dfrac{S_{i-1}}{L_{i-1}^{\cdot} + \Delta L_{i-1}}\, \Delta z_{i,\,i-1} + \dfrac{S_i}{L_i^{\cdot} + \Delta L_i}\, \Delta z_{i,\,i+1} = P_{z,i}
\end{cases}
$$

where

$$\Delta x_{i,i-1} = x_i - x_{i-1}$$

$$\Delta z_{i,i-1} = z_i - z_{i-1}$$

(x_i and z_i coordinates of the joint i)

and the like.

L_i^O, initial length of the element i

$$\Delta L_i = \frac{(S_i - S_i^O)\, L_i^O}{E \cdot A_i}\,,\quad \text{elastic lengthning of the element i}$$

S_i (S_i^O), final force (initial) present in the membrane element i

$P_{x,i}$ ($P_{z,i}$), component according to x(z) of the load on the joint i, addition of the contributions of the actions distributed on the elements i-1 and positive if directed according to increasing x(z).

The solution of the non linear system can be easily obtained proceeding iteratively by substitution, provided that in the various iterations the equation coefficients are changed both for the geometry variation and for the material characteristics of deformability, and that the load is adjusted to the geometrical varia-

tions in the structure and, in case of wind, to the variations in pressure coefficients, even if they are inferred from pressure distribution diagrams referred to stiff structures.

Obviously the operation shall be concluded when the equations (a) are solved, apart from a reasonably small predetermined percentage difference.

By using high speed computers, this solution method results remarkably easy, for the more frequent discretizations, and because it can write an equation at a time, without occupying wide sectors of the operative systems.

Moreover it must be observed that on the basis of this type of approach a complete cycle of interactive automatic projecting can be easily organized. In fact, (by means of a video-display) at the first stage of the study, it's possible to proceed to the research of the structure shape by optimizing it to the required functional purposes [1].

Given the geometrical conditions that must be respected by the pneumatic structure (giving for instance the coordinates x, e or z of some joints) and neglecting the proper weight, project values of the internal pressure can be chosen, as well as of the stress of the membranes structures (constant in the unconstrained membrane stretches) and of the length of the single elements L_i^o (basically n values of the ratio S_i^o/L_i^o are chosen).

The first solution of the equation system (a) indicates the shape of the structure [1]; this can be immediately adjusted to the project, submitting to the computer a new series of instructions.

Once the desired static configuration is reached, the effects of snow or wind loads can be analyzed by solving the equation system (a) again; in the same way the initial configuration can be rectified taking into account the proper weight.

2.3   Some applications

In order to exemplify what has been previously illustrated, same cases have been solved.

Figures 3 and 4 illustrate the geometry assumed by a pneumatic structure after having established, beside the length of its direction line, the conditions for the extremum points (1 and 21)

and, for example, a condition for an intermediate point (ex : the value of Z_8); in this case, owing to the internal pressure $p = 35$ Kg/m^2, the membrane stress is respectively $S_1 = 246$ Kg/m in the section from point 1 to point 8, and $S_{11} = 334$ Kg/m from 8 to point 21; in figure 3 is also indicated the deformed shape assumed by the structure placed over the initial one, without considering the material elasticity, due to a vertical load, such as snow, ($p_n = 15$ Kg/m^2) between point 11 and point 14; as the deformed shape clearly shows, the point 8 horizontally moves (in fact, it has been simulated the effect of a sliding hinge constraint).

Obviously, it results that the stresses decrease slightly in increasing the curve in the unloaded zone [$S_{min(1-8)} = 232$ Kg/m, $S_{min(8-21)} = 323$ Kg/m] .

Fig.4 shows, placed over the initial shape, the unextensional deformed shape under the effect of a load such as wind ($p_v = 60$ Kg/m^2) affected by coefficient $C(\theta)$ (being θ, in a generic point of the structure, the angle formed by the tangent to the membrane referred to the horizontal). These values, for exemplification purposes, have been desumed from the diagrams which are not strictly related (2). In this case : $S_{(1-8)} = 221$ Kg/m ; $S_{(8-21)} = 393$ Kg/m.

As an example, figures 5,6 and 7 give as a base the initial shape of a pneumatic structure, for which only the extremum conditions have been established : figure 5 showes the deformed states, with internal pressure of 35 Kg/m^2 under the action of a snow load having an intensivity $p_n = 25$ Kg/m^2, in the sector between joint 6 and joint 16, in the case of unextensible material as well as when it has been assumed the value of $5 \cdot 10^5$ Kg/m for the product E.s.

Figures 6 and 7 show the membrane deformation state under the action of a load due to wind having an intensity $p = C(\theta) \, p_v$, where p_v has been assumed equal to 80 Kg/m^2 and the function $C(\theta)$, according to some Authors [4] has been deduced from experiences on rigid models equally shaped and the material has been considered both as extensible and deformable with the values of E.s equal respectively to $5 \cdot 10^4$ Kg/m and $3 \cdot 10^4$ Kg/m.

In order to point out the influence of the material deformability also on account of the increase in the distribution of the pressure as a function of $C(\theta)$, it has been solved the case of a lowered structure (Fig.8), undergoing the same condi-

tions of internal pressure and of wind, still assumed $E.s =$
$= 5.10^4$ Kg/m.
In this case the influence of material deformability is considerable stronger as, while the initial membrane stress is $S_o =$
$= 703$ Kg/m, the final one for $E;s = \infty$ is $S = 2304$ Kg/m, which
slightly decreases, for $E.s = 5.10^4$ Kg/m, to $S = 2206$ Kg/m.

From the solutions of the above cases it is also possible to
plot useful diagrams giving, for example, the maximum movement
inward the membrane as a function of internal pressure values
p and p_v. Fig.9 shows the curve, with p = cost, giving the
maximum movement value inwards δ_{max} (referred to the width L
of the structure), as a function of p_v, for a structure having
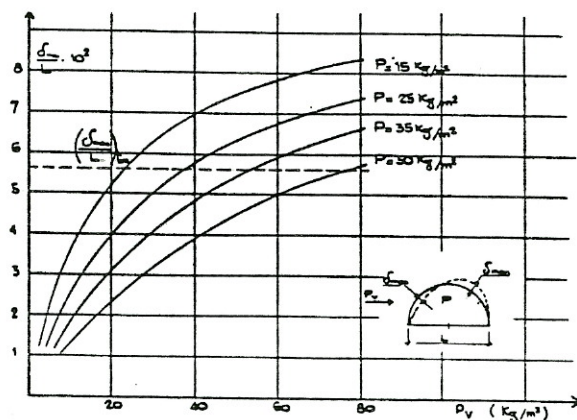the same initial shape of the one given in Fig.5.

Fig. 9 — Curve at p=cost which gives the value of the maximum
movement inwards δ_{max} (referred to the width L of
the structure) in function of pv.

Establishing the maximum acceptable value of the ratio δ_{max}/L,
it is possible to obtain the trend of p as a function of p_v (in
this case almost linear in the range of the practical values of
p). Such trend is useful to set out the project of a servo-me=
chanism automatizing the structure management. However, it must
be pointed out that the previously indicated limitations do not
consider particular turbolence of air currents which can also
bring about peculiar effects on the deformed structure.

2.4 Interactive calculation programm

For the solution of plane cases of pneumatic structures a program called CIL has been developped. This program operates on the computer CDC Cyber 76 of CINECA, through a graphic terminal Tecktronic 4014. The interactive computer aided design method adopted is shown in the following flow-chart.

In order to turn the connection more 'conventional', the program allows the modification of some parameters even during elaboration, thanks to a series of options.

In particular :
1) It is possible to modify the accuracy required in the determination of deformative states.
2) It is possible to combine in different ways the values of internal (p), of snow load (p_n) and of wind load (p_v).
3) It is possible to modify the values of the coordinates of the initial and final points of the structure cross section.
4) It is possible to constrain, both vertically and horizontally, any membrane internal point.
5) Once introduced the value of the material elastic modulus and of the section area of an unitary plane, the structure deformative state under load is calculated, considering the material deformability, by graphically placing it over the corresponding unextensible case.

Options combinations are obviously possible. It is therefore clear that the project stage is actually assisted by the automatic analysis, according to those modern trends which tend to set the engineer, relieved of heavy calculation burdens, at the centre of the synthesis work.

3.- THE SPACIAL CASE

The more general case of pneumatic structures with circular or square plan, stiffened or not by rope nets, can be takled, apart from the rather heavy methods from the calculation point of view [10], in the light of the procedure above illustrated for the plane case. Thus determining both the geometry of the initial state (i.e. only with the internal pressure), and the deformed state with external loads.
Consequently, after schematizing the structure with a quadrangolar mesh (3) lattice of fictitious rods for the generic joint k, the following equilibrium equations can be written, according to the axis of the general reference system :

b)
$$\begin{cases} \sum_i \dfrac{S_{k,i}}{L_{k,i}} \, \Delta x_{k,i} = P_{x,k} \\[2mm] \sum_i \dfrac{S_{k,i}}{L_{k,i}} \, \Delta y_{k,i} = P_{y,k} \\[2mm] \sum_i \dfrac{S_{k,i}}{L_{k,i}} \, \Delta z_{k,i} = P_{z,k} \end{cases}$$

the summations are extended to all the joints i surrounding the joint k, where

$$\Delta x_{k,i} = x_k - x_i$$

and the like

$$L_{k,i} = \sqrt{(\Delta x_{k,i})^2 + (\Delta y_{k,i})^2 + (\Delta z_{k,i})^2}$$

being

x_k, y_k, z_k the coordinates of the joint k

$L_{k,i}$ the length of the rod k, i

$S_{k,i}$ the force on the rod k, i

$P_{x,k}$ (and the like) the component according to x of the internal force applied on the joint k.

Having assumed suitable initial values of the stress density $q_{k,i}^0 = S_{k,i}^0/L_{k,i}^0$, for the fictitious rods, equal to the ratio between the stress of the rod and its length, the so linearized equation system (b) of 3n unknown values (coordinates of the non constrained joints) can be solved/through the iterative method, thus obtained a first configuration which can be easily controlled through graphic representation on cinescope. Proceeding then on the basis of the interactive automatic structural projecting approach, it is possible to rectify the initial shape by optimizing it in relation to prefixed purpose, also of functional type [11].
Moreover, working on the initial geometry, the continuous can be schematized with a quadrangolar mesh lattice; this, in or-

der to endow the rod system with a fictitious adjusting capability simulating the real deformative phenomena, because of which surface wrinkling phases occur in the presence of great movements.

Always through the system (b) after determining the new values of external load and introducing the value $(L_{k,i} + \Delta L_{k,i})$ instead of $L_{k,i}$ in order to take into account the elastic behaviour, the structure can be solved.

Such solution must however be always compared with an unelastic solution, and the final geometrical configuration must be then rectified according to the equivalent deformability of the fictitious rods (fig.10.a,b,c).

NOTATIONS

(1) From such geometrical configuration, the construction lengths can be inferred, subtracting the elastic longation due to the internal pressure from the length of the single elements.

(2) For the determination of the coefficients $C(\theta)$ reference was made to those of a circular directrix cylindrical membrane.

(3) At a first stage, a triangular mesh lattice can be chosen. Still, by adopting a lattice with quadrangolar meshes, the horizontal projection of which is represented by quadrila terals with almost parallel sides, it's easy to determine, in first approximation, the stress state of the membrane without resorting to heavy elaborations.

(4) We could, for instance, assume an uniforme stress state for the structure, even if discretized; but a square plan membrane with uniform stress state doesn't present substan tial characteristics from a utility point of view, especial ly with regards to the utilization of internal place close to the angles [11] .

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