ON THE ELASTIC INTERACTION BETWEEN ROPE NET AND SPACE FRAME ANCHORAGE STRUCTURES

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The main object of this paper is to illustrate the design procedure and computational methodology for pre-stressed rope net structures anchored to elastically collaborating edge structures. A first paper on the subject was presented in Ref 5. In the present paper the authors intend to examine deeply the interaction between the elastic anchorage system and rope net structures. The parameters involved in this comparative analysis are: 1) mesh net dimension, 2) mechanical elasticity of the anchorage system. It is important to observe that for some values of stiffness ratio between the anchorage and net structures is no longer possible to clear the actual mesh net in order to reduce the degrees of freedom and, consequently, the computational effort. The pre-stressing procedure, from slack cables, is simulated with the same algorithm proposed in the paper. This analysis permits the reduction of pre-stressing steps to a minimum with the relative saving in time and cost.

INTRODUCTION

In the field of structural roof design using a pre-stressed rope net anchored to elastically deformable edge structures (Fig 1), two problems are seen to dominate: numerical analysis by computer; optimal determination of some mechanical parameters so as to keep overall building costs to the minimum.

As regards the first problem, structural analysis of the rope net, elastically collaborating with the anchorage structures was faced according to the following schemes:

a) Continuous analysis of the net as a pre-stressed, anticlastic membrane; the edge anchorage structure is dimensioned with the forces transmitted by the membrane calculated with fixed end boundary conditions.

b) Analysis of the rope net with displacements permitted only
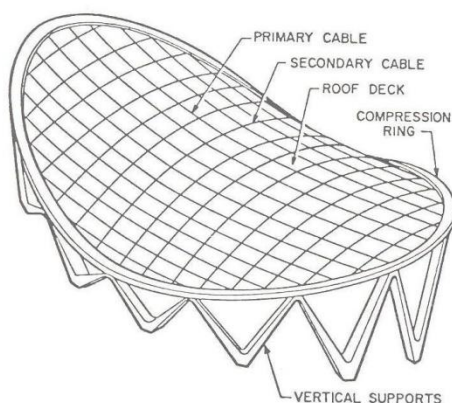

Fig 1

in a vertical direction (one degree of freedom per node). The real rope net is changed to an equivalent one, with ropes fewer in number, in order to reduce the number of unknowns. The elastic edge structure is analysed simultaneously to the net or by substructuring with the same method of structural analysis (displacement or force method).

c) Analysis of the rope net with three degrees of freedom per node. The edge and the net are analysed globally or by substructuring within the bounds of the same analysis method.

In agreement with the above, studies in Refs 1-4 were written. Definitions a) and b) introduce approximations that considerably restrict the field of structural investigation; considering fixed end constraints makes for an anti-economic solution; continuous membrane analysis is only possible for some analytically expressible surfaces and only for some load conditions. Consideration of rope nets contained in parallel planes is limitative, permitting no consideration of either more complex forms or loads which are not vertical, thus restraining the analysis to only those surfaces which are sufficiently flat. The definitions of structural analysis according to c) often lead to numerical problems principally manifested in the following ways:

- high conditioning number $C(K) = \lambda_{max}/\lambda_{min}$ of the matrix of the coefficients generated by the equilibrium method. The rope net is dominated by the slow vibration modes so the stiffness matrix, generated by the equilibrium method and expressed in constant-length computer words, reduces the information associated with the slowest vibration modes; as can be seen by expressing the stiffness matrix in terms of the eigenvectors and eigenvalues (Ref 6),

$$[K] = \sum_{i=1}^{n} \lambda_i \{V_i\} \{V_i\}^T \qquad \dots\dots 1,$$

λ_i = i-th eigenvalue,
V_i = the corresponding vibration mode.
To avoid truncation errors, especially when using computers that work with few bits, it is advisable to use the forces method in analysing the rope net. In this case we have

$$[K]^{-1} = \sum_{i=1}^{n} \frac{1}{\lambda_i} \{V_i\}\{V_i\}^T \qquad \dots\dots 2,$$

thus numerical truncation takes place on the stiffness vibrating modes;

- ill-conditioning of the coefficients matrix, both for the equilibrium method and that of the forces, due to the presence, in the case of global solution, of structural parts which are considerably stiffer than others. Ring stiffness, in the practical cases met with, especially with reinforced concrete rings, is considerably greater than that of the rope net, particularly in the direction of the normal at the structure surface. The finite elements, like beams with 6 degrees of freedom per node used for the edge structure as well as bar elements, monoaxially stressed, with three degrees of freedom per node, generate, for each degree of freedom, rows of the coefficients matrix numerically quite different. In the decay of the main diagonal during the Gauss solution procedure one often meets numbers near to machine zero. Similar situations are met with in

$$[K] = \begin{bmatrix} K_A & -K_A \\ -K_A & K_A + K_B \end{bmatrix} \qquad \dots\dots 3,$$

with $K_A \gg K_B$. $[K]$ is accurately represented only if K_B is not lost through truncation with respect to K_A;

- solution using the equilibrium method of structures with different behaviour in the geometric and material non-linearity field. The rope net has typical geometric hardening behaviour while the edge has geometric and material softening behaviour. The numerical effort is minimized by adopting the equilibrium method for structures with softening behaviour, while the forces method is more suitable in incremental solution of the hardening type of non-linearity.

As regards determination of the most suitable mechanical and structural parameters to consider in the design phase, the present paper aims to evaluate the influence of the following variables: (i) deformability of the anchorage structure; (ii) dimensions of the rope net mesh.

As for the elastic interaction between the net and edge during the pre-tension and "0" state phase, the technical literature makes no questions. Some considerations on this problem are made alongside illustration of the numerical examples.

NOTES ON THE ANALYSIS METHOD BY FUNCTIONAL SUBSTRUCTURING

The method adopted for structural analysis has already been illustrated in a general manner in Ref 6. As for the specific case, the subject of this paper, the following hypotheses are made:

- mixed analysis for functional substructuring;
- softening material edge structure analysis, small displacements and deformations by means of the equilibrium method;
- analysis of the rope net in geometric hardening, large displacements and small deformations by the forces methods;
- iterative analysis by elementary physical substructuring of each rope withn the rope net structure.

Figure 2 illustrates the substructuring carried out, where:

$S_{I;1}$ = edge substructure;
$S_{II;i}$ = net substructure formed of F substructures elementary;
i = 1-F ropes.

It is thought that we have now obtained, with one of the methods indicated in Refs 5,6 the solution of state "0" and, therefore, to have noted:

$(X^o;Y^o;Z^o)_j$; j = 1-n = "0" state coordinates of the nodes,
L^o_{kj} ; $\overline{kj}$ = 1-m = "0" state length of the bars,
S^o_{kj} ; $\overline{kj}$ = 1-m = "0" state stress in the bars,
P^o_j ; j = 1-n = "0" state load in the bars.

Let us now consider I and II interconnected in the set of points B , see Fig 2.
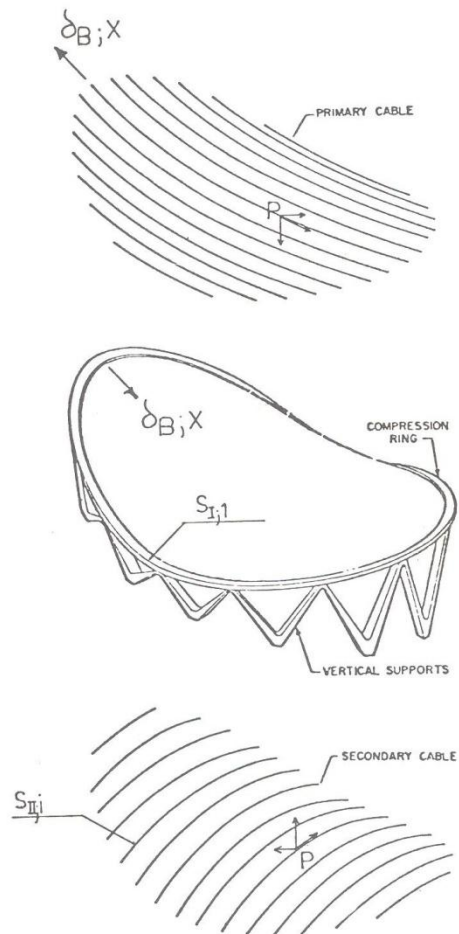
By separating the displacements, one gets:

Fig 2 Functional substructuring

$$\begin{bmatrix} K_{AA} & K_{AB} & \vdots & 0 \\ K_{AB}^T & K_{BB} & \vdots & G_B \\ \cdots & \cdots & \vdots & \cdots \\ 0 & -G_B^T & \vdots & F \end{bmatrix} \begin{Bmatrix} \delta_A \\ \delta_B \\ \cdots \\ X \end{Bmatrix} = \begin{Bmatrix} P_A \\ P_B \\ \cdots \\ \delta \end{Bmatrix} \qquad \cdots\cdots 4,$$

where: K = stiffness matrix of substructure I relative to the A (internal) and B type boundary nodes; F = flexibility matrix of the substructure II; G = coupling matrix between I and II; δ_A; δ_B = displacement vectors in I; X = indeterminate and boundary forces; P_A; P_B = load term in I; δ = displacement term in II.

The solution to Eqn 4 can be obtained in displacement terms by considering:

$$\begin{bmatrix} K_{AA} & K_{AB} \\ K_{AB}^T & K_{BB}^* \end{bmatrix} \begin{Bmatrix} \delta_A \\ \delta_B \end{Bmatrix} = \begin{Bmatrix} P_A \\ P_B^* \end{Bmatrix} \qquad \cdots\cdots 5,$$

with:

$$K_{BB}^* = K_{BB} + G_B F^{-1} G_B^T \qquad \cdots\cdots 6,$$

and

$$P_B^* = P_B - G_B F^{-1} \delta \qquad \cdots\cdots 7,$$

where K_{BB}^* and P_B^* are interpreted respectively as the stiffness matrix and load vector, transformed by the effect of the common node displacements, and I and II of type B, equivalent to static condensation of substructure II.

In the non-linear field, where it is necessary to act interactively and/or incrementally, Eqns 5, 6 and 7 suggest adoption of a sequential relaxation iterative procedure which includes the following phases:

a) Definition of the boundary forces in II.

b) Evaluation of first-attempt $P_B^{*(1)}$, Eqn 7.

c) Evaluation of first-attempt $K_{BB}^{*(1)}$, Eqn 6.

d) Solution of Eqn 5.

e) Return to a).

f) Convergence check.

As regards phase a), solution of the net and, therefore, systematic preparation of the $\boxed{F}$ and $\boxed{G_B}$ takes place following the method indicated at point 4.2 of Ref 6.

Without taking anything away from the generality of the method, a first value of the forces in the net may be obtained by exploiting the physical, hypostatic characteristic of the net. Considering the net to be formed by m bars and n nodes one has, in general:

$$3n > m \qquad \cdots\cdots 8.$$

The equilibrium conditions

$$\boxed{A} \{S + \Delta S\} = \{P + \Delta P\} \qquad \cdots\cdots 9,$$

where: $\boxed{A}$ = matrix of the cosinus directors by undeformed geometry; $\{S^\circ + \Delta S\}$ = vector of the forces on the bars after load variation; $\{P^\circ + \Delta P\}$ = load vector, present an $\boxed{A}$ rectangular matrix with 3n rows and n columns.

A very useful first-attempt value, to diminish the iterative cycles, can be obtained by the least squares method, minimizing the non-equilibrated load residue:

$$\begin{cases} \boxed{A} \{S^\circ + \Delta S\} - \{P^\circ + \Delta P\} = \{R\} \\ \{R\}^T \{R\} \longrightarrow \min \end{cases} \qquad \cdots\cdots 10.$$

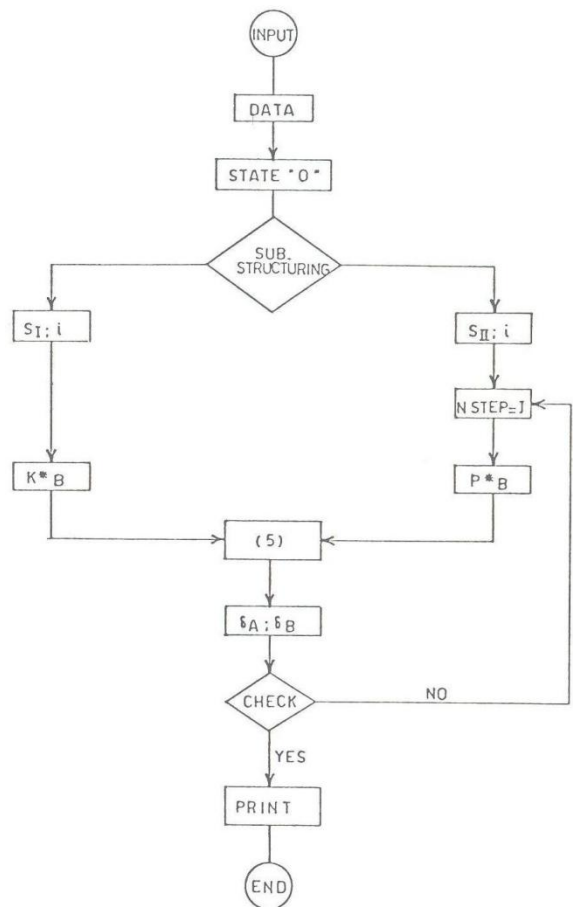The flow diagram, in accordance with the above, is synthesized in Fig 3.

Fig 3

NUMERICAL EXAMPLES

According to the analysis method illustrated section above, we intend to examine a rope net structure anchored to an elastic space frame, loaded with a vertical uniform load of 800 KN/M^2.

The rope net adopted is of variable mesh net and the edge structure is a ring space frame which geometry is defined as the intersection of a hyperbolic paraboloid surface with a circular cylinder of 60 m diameter, analytically described by:

$$\left[\begin{array}{l} \dfrac{X^2}{40,909} - \dfrac{Y^2}{64,282} = Z \\ \\ X^2 + Y^2 = 900 \end{array} \right. \qquad \dots \dots 11.$$

The variation of the mesh net dimension is illustrated in Fig 4.

The initial stress state (STATE "0") in the rope net is defined, in function of the mesh net dimension (i) as:

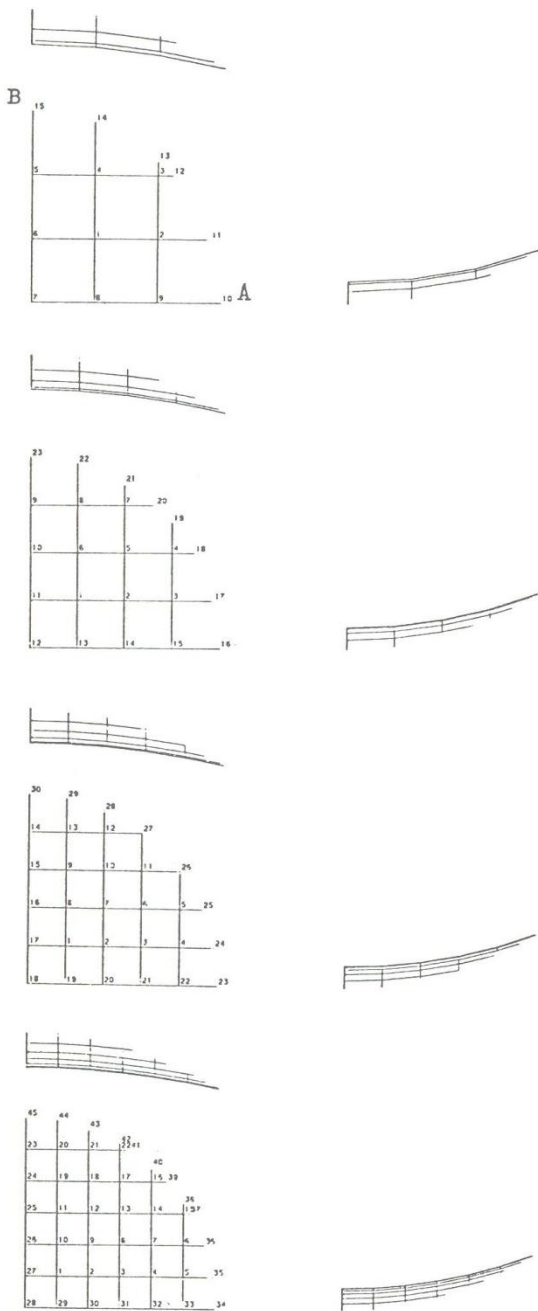$$H_p = 4000 \cdot i \; ; \quad H_s = 6286 \cdot i$$

where: H_p = horizontal component of carrying rope forces;
H_s = horizontal component of stabilizing rope forces.

The anchorage structure is represented by a space frame only vertically supported in discrete joints.

The area and inertia parameters of the cross section of the ring structure are indicated in Table 1.

Fig 4   Mesh-net configurations
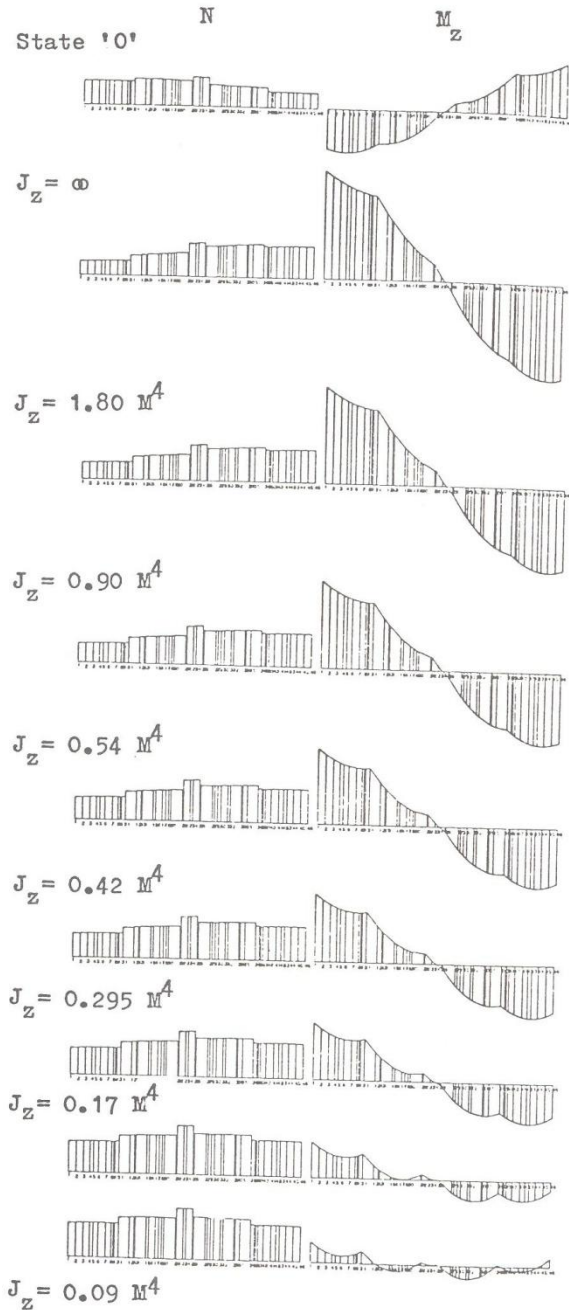
Fig 5   N and M_z distribution along a quarter ring (AB) in function of J_z variation for a 10x10 M, mesh-net.

Table 1

J_Y	J_Z	A
∞	∞	∞
0.34	1.8	0.9
0.17	0.9	0.5
0.102	0.54	0.2
0.079	0.42	0.16
0.056	0.295	0.11
0.032	0.17	0.06
0.017	0.09	0.03

With the above mentioned structural data is our intention to analyse the influence of the elastic collaboration between rope net-anchorage structure and the mesh net dimension in order to evaluate the variations in the state of stress and deformation of the global structure.

In Fig 6 is illustrated the variation of the vertical displacement in the center of the rope net in function of the inertia parameter J_Z of the ring section. The displacement for fixed end displacements ($J_Z = \infty$) in the rope net is also indicated.

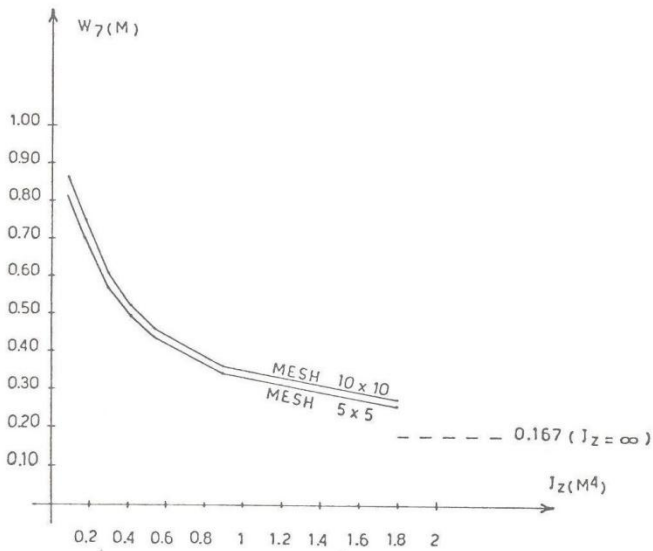Fig 7 Horizontal displacements of ring structure

Fig 6 Vertical displacement of central node in rope net

In Fig 7 the horizontal displacement variations of section A and B of the ring structures are drawn.

Figure 8 shows the variation of the principal bending moments (M_Z) in the section A of the ring structure in function of J_Z for all the mesh-net dimensions here considered.

Figure 9 illustrates the same for the axial force in section A.

Figure 10 shows the typical variation of cable forces for the mesh of 10 x 10 m, the state "0" initial values are also indicated.
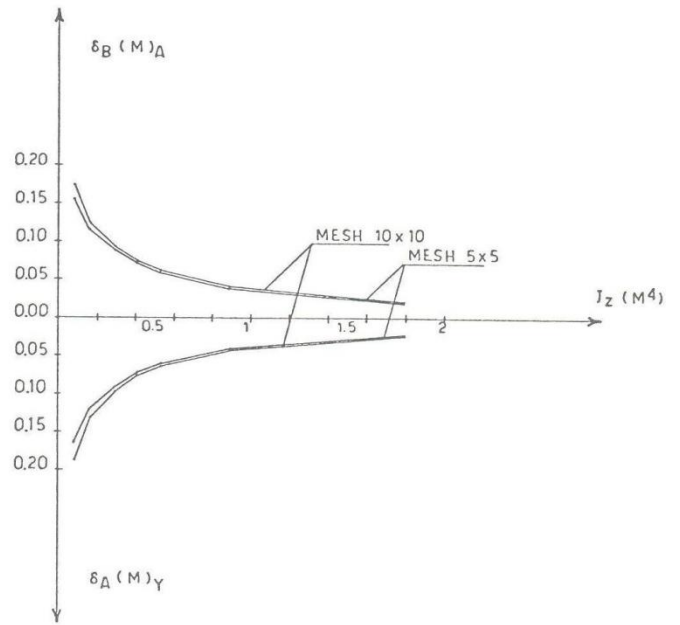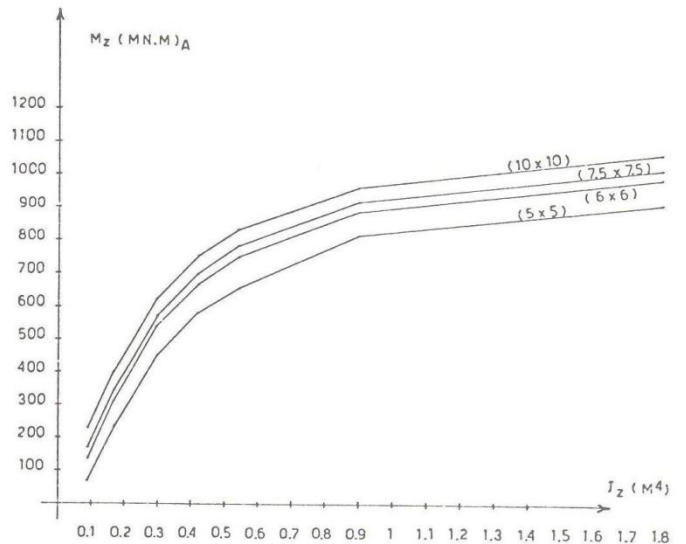

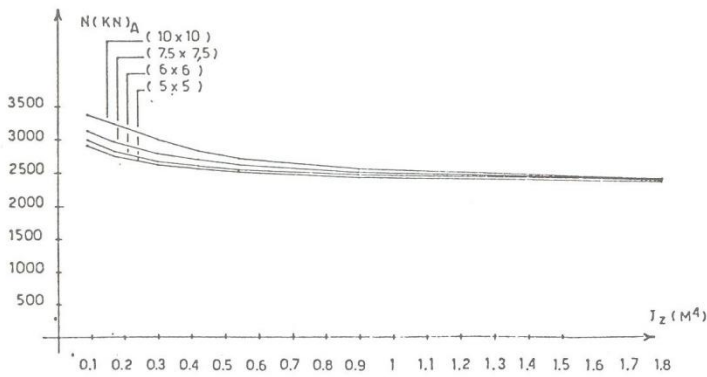Fig 8 Bending moments in section A of the ring

Fig 9 Axial forces in section A of the ring
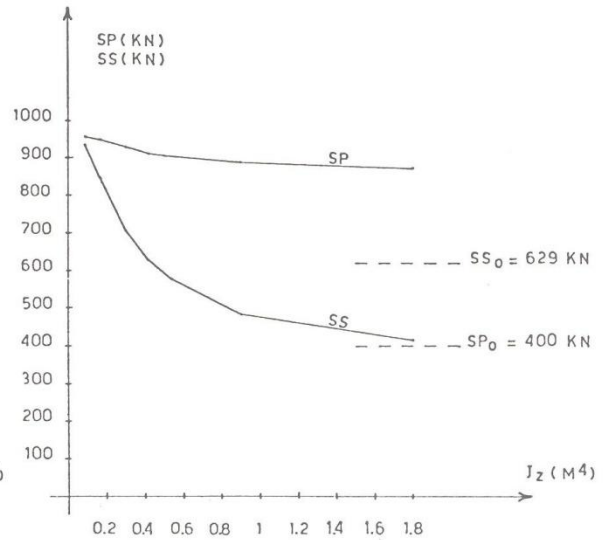
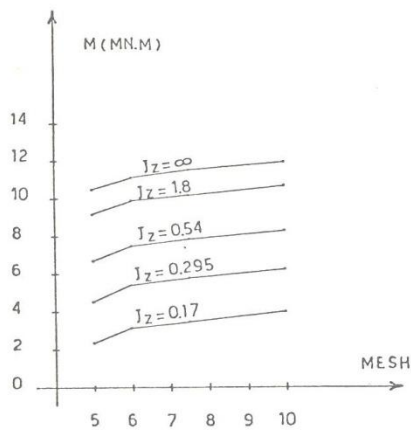Fig 10 Cable forces in section A   and B

Fig 11 Moment variation in section A of the ring in
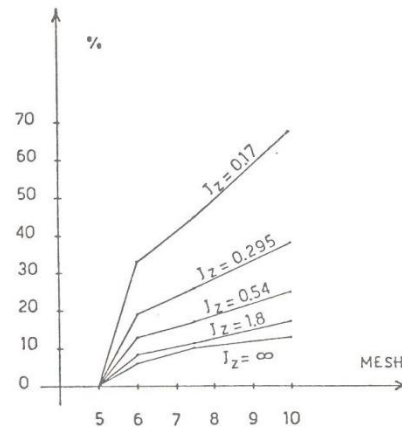   function of the mesh

Fig 12 Percentual moment variation in function of mesh-net

Figures 11 and 12 show clearly the moment variation in
function of the mesh net.

Figure 13 gives the variation of displacements in section A
and B of the ring structure considering the elastic in-
teraction between ring and rope net during the state "0"
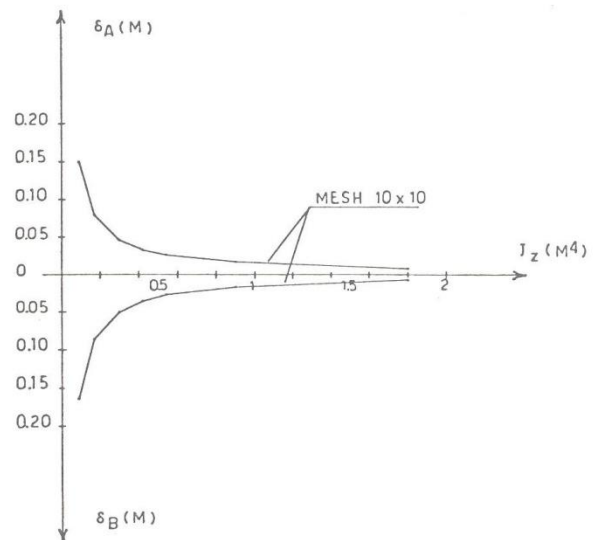research.

Fig 13 Displacements in section A   and B of the ring
        in pre-stressing state.

CONCLUSIONS

From the examination of the plotted results of Figs 6 to 13 it is possible to conclude that:

- the fixed end hypothesis ($J_Z = \infty$) don't give any useful design information for actual structures, even for a pre-liminary design phase;

- during elastic interaction, the stresses in the anchorage structures decrease drastically specially for low values of J_Z (see Figs 5 and 8). In order to find a minimum for the structural cost it is very important to examine the variation of the bending moments in function of the relative stiffness of both structures, rope net-anchorage frame. For the structure analysed in this paper the suitable inertia range is found between 0.2÷0.5 where the decay of M_Z is more than proportional. Lower bounds of J_Z are imposed due to the deformative limitation in the rope net or for maximum displacements in the anchorage frame (Figs 6 and 7);

- the anchorage frame displacement gives to the global struc-ture a self-adjusting property because for a certain value of inertia (in our case 0.4; Fig 10) the "natural" decreasing forces in the stabilizing ropes return to increase, as re-gards as the state "0" value (SS_o), displacements forced by the elastic interaction between the rope net and the anchor-age structure;

- the moment distribution in the anchorage frame changes evidently due to the variation of the mesh-net dimension, see Figs 11-12. The choice of the mesh-net is very impor-tant in order to minimize the structural global cost. This cost is evidently related to the cost of the roof covering system. From Fig 12 is possible to observe that from a mesh of 10x10 we find a 67% of moment increase;

- Figure 13 shows the displacements of the anchorage struc-ture for the fixed state "0 forces. The elastic interaction in state "0" phase must be considered during the working project of the roof structure in order to control the pre-stressing operations.

The results obtained suggest further investigations, consi-dering the influence of other structural parameters as:

- variation of sag and curvature ratio;

- variation of the initial pre-stressing state;

- variation of the total difference level between higher and lower anchorage points (total curvatures).

Results on the matter will appear in a further publication.

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