OPTIMAL DAMPING SYSTEMS FOR FLEXIBLE FOOTBRIDGES

Nicola COSENTINO
Structural Engineer
Bologna, Italy

Massimo MAJOWIECKI
Professor
IUAV University
Venice, Italy

Chiara UTILI
Structural Engineer
Bologna, Italy

Summary

Pedestrian footbridges are characterized by a low inherent damping. Hence, to avoid comfort problems it is often necessary to add external damping to the structure. In this paper, two different damping strategies have been analyzed. The first one consists in placing point dampers between the ground and the bridge deck, close to the bridge ends. The second one in based on the idea of using non-structural elements to add damping to the structure; in the present case, a dissipative handrail has been hypothesized.

The damping system performances have been evaluated in terms of modal damping ratio within the frequency range interested by common vertical excitations. Point dampers create a non-diagonal modal damping matrix; thus, the system is non-classically damped and damping optimization problems arise. In the case of distributed dampers, as the dissipative handrail is, the modal damping matrix is approximately diagonal and a “classical damping” can be hypothesized; the modal damping increases as the damping coefficients increase.

The performed analyses show that the point dampers exhibit good performances on the first few modes. On the other hand, the dissipative handrail offers good performances on higher modes but is not able to damp the first ones. Subsequently, the best solution is recognized in the joint use of the two considered systems: point dampers at the bridge ends and dissipative handrail. The procedure to analytically evaluate the optimal parameters for both the systems is outlined and an application to a suspension footbridge is presented.

Keywords: Footbridge, pedestrian action, vertical vibrations, damping, optimization methods.

1. Introduction

Footbridges are getting common structures in the landscape nowadays. Since they are “lively” constructions, due to their growing flexibility, they need peculiar cares. The use of modern materials and technologies allows the construction of very light structures with respect to the live sustainable loads. Furthermore, the nature of the usually used materials (steel, composite fibres, etc.) and the high level of working stresses, give rise to very small inherent damping values. Finally, the main frequencies of such flexible structures are almost always included in the human walking frequency range (1,5 to 5 Hz approximately). This means that flexible pedestrian bridges are commonly subjected to dynamic effects induced by people moving across them. Increasing damping is usually the only way to reduce the pedestrian induced vibrations. These vibrations, if they are not suppressed or strongly mitigated, can give rise to fatigue problems and damages in structure components. Since to modify the inherent damping is quite difficult, it gets necessary to act on external damping.

Commonly, in order to mitigate the vibrations, stiffening systems or damping devices (as, for instance, TMD and visco-elastic dampers) are placed. When visco-elastic dampers are used, one of the design problem is to determine the optimal value for the damping coefficient. The use of external dampers requires structural movements with respect to the ground (or other stiff structures) or internal deformations. Hence, theoretically, each displacement or deformation location represents a possible damper location. In the paper, some optimization criteria, oriented to both the choose of the damper location and the evaluation of the best damping coefficient, are examined.

Actually, viscous damping is considered only, due to its wide use, applicability, advantages and the possibility to simulate other damping typologies. In the present work, an alternative approach is proposed and analyzed. It will be proved that, if an appropriate distributed dissipative system is used together with lumped visco-elastic dampers, the global system performances increase in terms of both the covered frequency range and the benefit-cost ratio.

2. Existence of an Optimal Damping Value

The existence of an optimal value for the damping coefficient was demonstrated, for cables, by Kovacs [1]. In fact, increasing damping over a certain threshold can induce a fictitious stiffness in the structure which vanishes the energy dissipation role of the damper. The concept can be applied to footbridges as well, as it is clearly shown in Fig. 1 by mean of the limit conditions $c = 0$ e $c = \infty$: of course, if $c \to 0$ no damping is added to the structure; on the other hand, if $c \to \infty$ the damper becomes a rigid restraint for its application point and the structure simply changes its modal frequencies and shapes, without dissipate energy.

Following the approach outlined in [2] for taut cables, the optimal damping evaluation procedure and the damping curves are derived for the case of mixed dissipative systems on footbridges.

3. Dynamic Equations and Eigenvalue Problem

An accurate evaluation of dampers performances, that is the evaluation of the structural response in presence of external damping devices, is often quite cumbersome and it usually requires time domain analyses (Monte Carlo procedure), due to the induced non-linearities. Nevertheless, a preliminary evaluation of the external damping system efficiency can be performed by mean of a relatively simple orthogonal decomposition technique [3]. This is very useful during the preliminary design stage, when different solutions must be compared to recognize the best damper locations and the optimal damping coefficient.

The procedure substantially consists in assuming a linear viscous behaviour for the dampers (eventually approximating other constitutive laws) and in evaluating the damping ratio relative to the few complex modes of the non-classically damped structural system which are significant in terms of dynamic excitation sensitivity. The evaluated damping ratios can then be used to evaluate the resonant response. The damping matrix **C** is initially assembled in the original coordinate system, taking into account the external damper contributions. In general, the system will be non-classically damped, since the Caughey-O'Kelly condition [4] does not hold:

$$\mathbf{C}\mathbf{M}^{-1}\mathbf{K} \neq \mathbf{K}\mathbf{M}^{-1}\mathbf{C} \qquad (1)$$

where **M** and **K** are the mass and the stiffness matrix respectively. Under these conditions, the classical modal analysis of the undamped system can still be performed, but the corresponding modal equations are not uncoupled. However, the natural modal coordinates can be used as a simplified orthogonal base to approximately investigate the behaviour of the non-classically damped complex modes. With this purpose, it is assumed that a given number *n* of undamped modes are still adequate to represent the externally damped structural response in terms of both dynamic loading process and damper location movements. Analytically, let

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{0} \qquad (2)$$

be the free vibration equation of motion in the original Lagrangian space. By projecting it on the undamped modal space, the equation system becomes:

$$\mathbf{M}^*\ddot{\mathbf{b}}(t) + \mathbf{C}^*\dot{\mathbf{b}}(t) + \mathbf{K}^*\mathbf{b}(t) = \mathbf{0} \qquad (3)$$

where $\mathbf{M}^* = \mathbf{M}^T\ \Psi\ \mathbf{M}$, $\mathbf{C}^* = \mathbf{C}^T\ \Psi\ \mathbf{C}$, $\mathbf{K}^* = \mathbf{K}^T\ \Psi\ \mathbf{K}$ are the undamped modal mass, damping and stiffness respectively. Since $\mathbf{C}^*$ is not diagonal, the Eq. (3) is not uncoupled. The free vibrations of the externally damped structure can be determined by assuming that the system in Eq. (3) admits the solution:

$$\mathbf{b}(t) = r \cdot e^{\alpha t}\boldsymbol{\psi}^* \qquad (4)$$

If the Eq. (3) is rewritten by using Eq. (4), the following eigenproblem is found:

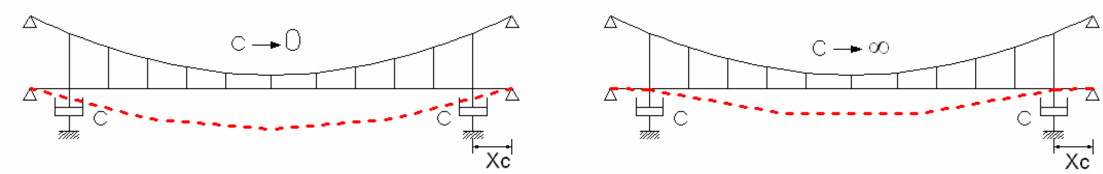

Figure 1. The existence of an optimal damping value for flexible structures.

$$\left(\alpha^2 \mathbf{M}^* + \alpha \mathbf{C}^* + \mathbf{K}^*\right)\boldsymbol{\psi}^* = \mathbf{0} \tag{5}$$

and it gives rise to eigeinvalues α and eigenvectors $\boldsymbol{\psi}^*$ which are, in general, complex (if the damping matrix is non-classical). The eigenproblem in Eq. (5) can be solved by means of standard algorithms as well. If, in the vibration shape expressed by the Eq. (4), it is stated:

$$\alpha_i = \beta_i + \gamma_i \sqrt{-1} \tag{6}$$

it results in:

$$\omega_i = \sqrt{\beta_i^2 + \gamma_i^2} \quad \text{and} \quad \xi_i = -\frac{\beta_i}{\omega_i} \tag{7}$$

ξ_i being the i^{th} required complex non-classical modal damping ratio.

The proposed procedure allows a drastic reduction of the computational effort with respect to the direct evaluation of ξ_i in the original Lagrangian space. In fact, only the first few complex eigenvalues are necessary, because only the first few structural modes are usually sensitive to the dynamic action. Let *h* be the number of these required damping ratios; *n* the number of undamped modes necessary to represent the externally damped structural response in terms of both dynamic loading process and damper location movements; and *N* the original Lagrangian degrees of freedom of the structure. Surely, *n* has to be sufficiently larger than *h* but usually, for flexible structures, *n* is much smaller than *N*. Since only the first *n* undamped eigenvectors are necessary in writing the system in Eq. (3), its dimension is much smaller than the original system in Eq. (2). Hence, the solution of the eigenproblem of Eq. (5) is much faster than the Eq. (2) corresponding one. Such a simplification assumes great importance when parametric studies are involved, as it is likely to occur when optimal damping coefficients and/or damper positions are investigated, through the parametric scan of the variable ranges.

4. Simply Supported Beams

The case of a flexible simply supported beam with two dampers located at its ends is treated firstly. As a matter of fact, as it will be shown below, for such a footbridge typology the optimal damping problem is just a theoretical task, the available damping level being always sufficient in the interested frequency range. Nevertheless, the presented case points out both some numerical convergence aspects and some qualitative considerations in damping flexible decks. The problem is schematized in Fig. 2, where *EJ* is the beam bending stiffness, *m* is the mass per unit length, *L* the total beam length, *C* the dampers coefficient, (*x*,*y*) the Lagrangian coordinates.

If modal shapes are approximated by sinusoidal functions:

$$\psi_i(x) = \sin\frac{\pi i x}{L} \tag{8}$$

i being the mode number, simple analytical steps allow to obtain the undamped modal eigenfrequency:

$$\omega_i = \left(\frac{\pi i}{L}\right)^2 \sqrt{\frac{EJ}{m}} \tag{9}$$

If the non-diagonal terms of the modal damping matrix are neglected, that is the “non-classic” damping contribute is not considered, the modal damping ratio can be simply obtained as:

$$\xi_i = \frac{C_{ii}^*}{2\sqrt{K_{ii}^* M_{ii}^*}} = \frac{2CH^2}{L\sqrt{mEJ}} \tag{10}$$

that is, the damping ratio is independent from the mode number and it is linearly increasing with the damping *C*.

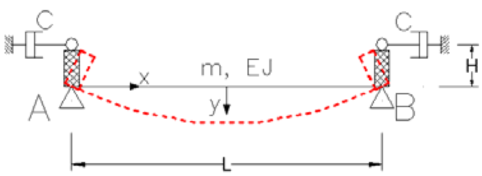

Figure 2. Scheme of the simply supported beam with end-located dampers.

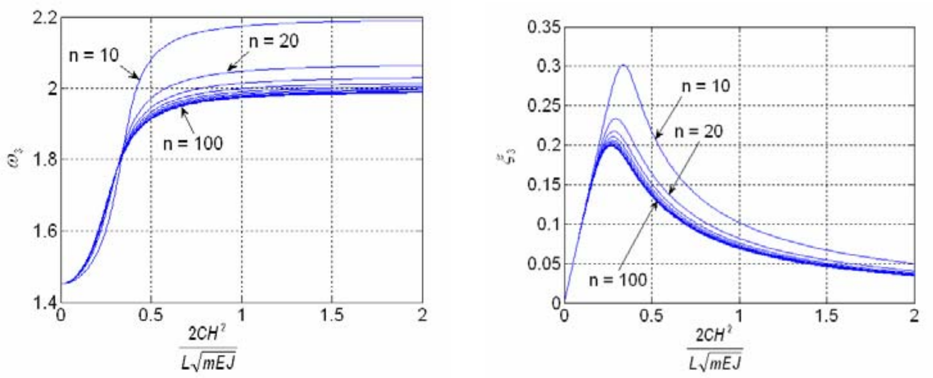

Figure 3. Numerical convergence of modal frequency and damping ratio of the 3rd mode by using different undamped modes.

This implies that the existence of an optimal damping ratio can be pointed out only if the full damping matrix and the subsequent complex modes are considered.

In the presented example, the beam parameters have been set to: m = 6 kg/m; EJ = 160200 N m^2; L = 100 m; H = 1 m.

Since for real structures, the number of available undamped modes is limited, the first step is to evaluate the number of undamped modes which are necessary to adequately describe the effective damped vibration. At this purpose, the complex non-classically damped modes have been evaluated by using different number of undamped modes n. Fig. 3 shows the results in terms of modal frequency and modal damping ratio relative to the 3rd damped mode.

Fig. 3 clearly shows the existence of an optimal damping value, in terms of maximum modal damping ratio and the stiffening effects of “strong” dampers, pointed out by the frequency shift which arises by moving from small to large C values. Furthermore, it can be noted that the behaviour of the considered non-classical mode is well kept by the first 20÷30 undamped modes. Similar analyses, carried out for other modes, show that the behaviour of the first 15 non-classical modes is sufficiently represented by the first 60 undamped modes. Subsequently, n = 100 undamped modes are used to investigate the behaviour of the first 10 actual complex modes. Results are shown in Fig. 4, in terms of modal frequency and modal damping ratio relative to the first 10 damped modes.

It can be observed that the maximum available modal damping ratio is very large for the first few modes but it decreases for the higher ones. Moreover, if such higher modes need to be damped, the corresponding optimal C value moves toward small magnitudes, so conditioning also the available damping for the lower modes. This aspect is confirmed by analyzing the stiffening effect of a “strong” damper on the different modes, as pointed out by Fig. 4(a) in terms of induced frequency shifting.

Actually, in simply supported beam footbridges, even when they are flexible, only the first few modes may need to be externally damped, due to resonance with pedestrian actions. In fact, the frequency of higher modes is far from the excitation range. Nevertheless, the results pointed out by the presented example, in addition to the convergence considerations and the evaluation of the modal damping itself, show that higher modes of suspension or cable stayed footbridges, which are strongly affected by the bending stiffness of the deck, may be difficultly damped by end-located dampers: since in these footbridge typologies the pedestrian exciting frequency range usually concerns numerous modes, this is not a fictitious problem.

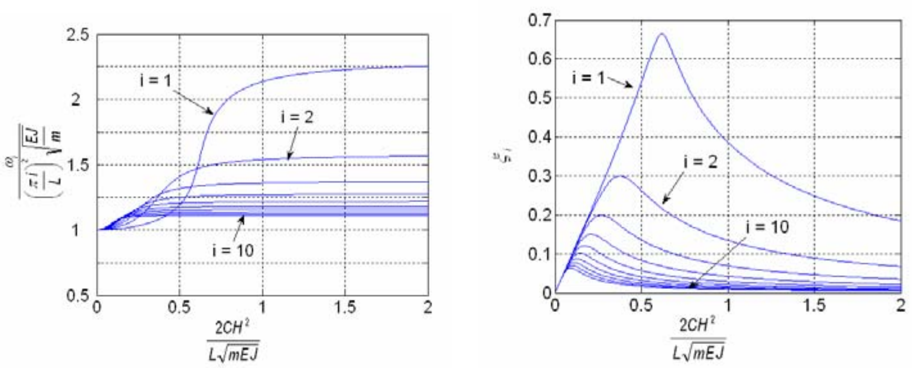

Figure 4. Modal frequency and modal damping ratio of the first 10 real modes for different values of the damping coefficient.

5. Suspension Footbridges

Suspension footbridges are among the most sensitive to the pedestrian dynamic excitation in the vertical plane direction. In fact, due to their very flexible decks (deck bending stiffness is not necessary to the equilibrium) they are usually characterized by the presence of numerous vibration modes within the range of the most frequent pedestrian induced vertical vibrations (commonly 1.5 to 3 Hz).

In order to keep some general features of the analyzed damping systems and to be able to check the results by mean of time domain integration of the full motion equations, a simplified suspension footbridge model has been considered. A vertical in-plane FEM model (one suspension cable, the corresponding deck girder and the hangers) has been realized. Different damper arrangements have been studied, compatibly with technically possible solutions.

5.1 The model description

The analysed mathematical model, schematized in Fig. 5(a), consists in a suspension footbridge with a 100 m main span length and a sag/span ratio of 1:8. The main cable is 70 mm diameter and the deck girder is made of a HE100A profile. A total amount of 39 hangers, 20 mm diameter, have been used. The deck is supposed to be 4 m width; so the vertical in-plane model supports the 2 m width half deck. The cable Young modulus is 165000 MPa and the deck girder one is 210000 MPa. The dead loads are supposed to be 2 kN/m^2; live loads of 4 kN/m^2 have been used for the static dimensioning. Finally the main cable pretension is 400 kN.

The modal analysis has been carried out by assuming the geometric stiffness matrix corresponding to the pretension and the dead loads. The first 100 undamped modes have been evaluated; the first 20 modes are characterized by a frequency of 0.35 to 3.87 Hz, which is approximately the frequency range where additional damping could be necessary. The modal shapes have been normalized to obtain: $\mathbf{M^*} = \mathbf{M}^T\ \Psi\ \mathbf{M} = \mathbf{I}$ and $\mathbf{K^*} = \mathbf{K}^T\ \Psi\ \mathbf{K} = \mathrm{diag}\,(\omega^2_i)_{i=1,n}$.

5.2 Dampers arrangement

Four damper arrangements have been considered. The first three and most typical solutions consist in locating point dampers between the deck and the ground, close to the bridge ends, corresponding to the first, the second or the third hanger, respectively. The fourth one is based on the idea of using non-structural elements to introduce additional damping in the structure: in the present case, a dissipative handrail has been hypothesized.

In the first typology case, classical - and often quite expensive - linear viscous dampers have to be used: in fact, as it will be shown below, the system performances are really sensitive to the device parameters which, hence, have to be sufficiently stable. In the second one, the cited sensitivity is much smaller. Hence, more rough and economical systems can be adopted; even non-linear and temperature sensitive devices can be used as, for instance, moving tubes filled with high damping rubber material. Nevertheless, in the present study, linear viscous damping in considered only: it can be seen as the equivalent viscous damping for real systems. Fig. 5(b) shows a scheme of the different damper arrangements: the arrangement C_1 consists of two point dampers located in correspondence of the first and last hangers; similarly, C_2 and C_3 consists of dampers located in correspondence of the seconds and thirds hangers; finally, the arrangement C_d consists of a series of dampers placed between all the hangers at the handrail level, linked to the deck girder by sufficiently stiff elements.

5.3 Modal damping features

In the present case, the damping system performances are evaluated in terms of modal damping ratio within the frequency range 0 to 3 Hz. As a matter of fact the frequencies lower than 1.5 Hz are rarely excited by ordinary pedestrian vertical actions but they can be intentionally excited by "vandalism jokes". Firstly, it is important to understand how each one of the examined systems behaves in terms of modal damping matrix $\mathbf{C^*} = \mathbf{\Psi}^T\ \mathbf{C}\ \mathbf{\Psi}$.

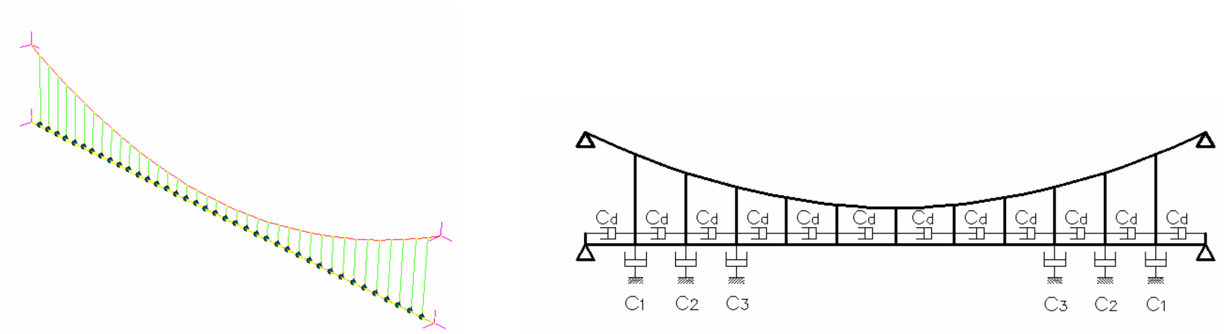

Figure 5. Scheme of (a) the FEM model used for the modal analysis and (b) the four considered damper arrangements.

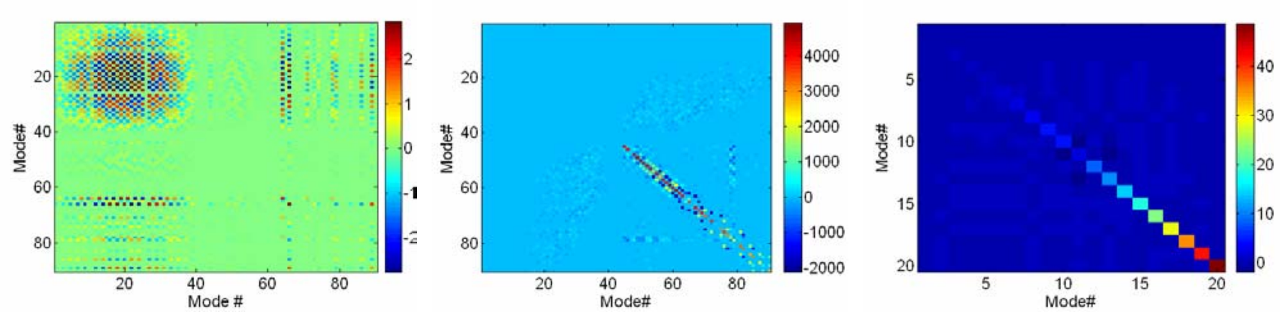

Figure 6. Modal damping matrix relative to the case (a) C_1 = 30 kN s/m; (b) and (c) C_d = 60 kN s/m.

Fig. 6(a) shows the modal damping matrix relative to the case of point dampers below the first hanger at the two bridge ends (case C_1). The first 90 undamped modes are considered: higher modes are strictly related to local vibrations of cables. It can be noted that such a matrix is completely sparse, with a strong interaction between the different modes. The resulting damping is strongly "non-classic". The system performances can be suitably evaluated only by taking into account the modal coupling and, hence, by solving the complex eigenvalue problem, as stated in the paragraph 3. The cases of point dampers below the second or the third hanger at the two bridge ends (cases C_2 and C_3) give rise to very similar considerations.

Fig. 6(b) shows the modal damping matrix relative to the case of a dissipative handrail and Fig. 6(c) shows a zoom of such a matrix on the first 20 modes which are within the interested frequency range. In this case, it can be noted that C* is substantially diagonal. Hence, the modal damping can be approximately evaluated as for classically damped systems.

5.4 Performances of the different damper arrangements

Following the above outlined properties of the modal damping matrices, the performances of the different examined systems can be now evaluated in terms of modal damping ratio. As regard the "dissipative handrail", since the modal damping matrix is approximately diagonal, the modal damping ratio can be evaluated as:

$$\xi_i = \frac{C^*_{ii}}{2 M_{ii}\, \omega_i} \qquad (11)$$

As it is pointed out by the Fig. 6(c), the modal damping coefficients increase as the mode numbers increase and vice-versa. Thus, this system is not appropriate do damp the first modes: too large damping coefficients are necessary to do it; dampers result over-dimensioned for higher modes; the benefit-cost ratio decreases considerably; the technical feasibility becomes difficult. This is clearly shown in Fig. 7(a), where the modal damping ratio of the first 20 modes, evaluated by mean of Eq. (11), is reported for a given value of the distributed damping coefficient.

The performance of the deck-end located dampers is evaluated by solving the complex eigenvalue problem. Modal (complex) damping ratios have been calculated for different values of the damping coefficient C_i and for all the modes whose frequency is within the range 0 to 3 Hz (20 modes). In Figs. 7(b) and 7(c), the results corresponding to the dampers located below the first (C_1) and the third (C_3) hangers are reported, respectively. It can be observed that the available modal damping ratio is quite small, especially if all the modes are simultaneously considered. This is basically due to two circumstances: firstly, the damping coefficient value which is optimum for a given mode is not optimum, in general, for other modes; secondly, the higher modes are not efficiently damped by the end-deck-located dampers, as it was already foreseen by analyzing the case of a simply supported beam.

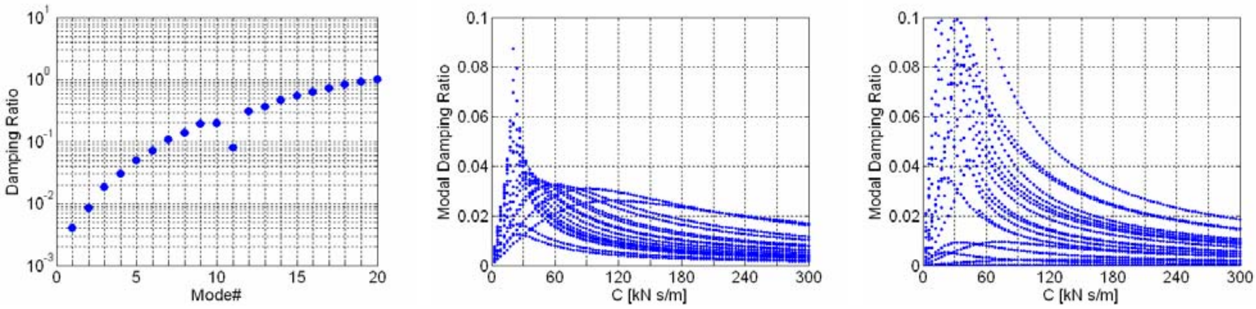

Figure 7. Modal damping ratio of first 20 modes for (a) C_d = 60 kN s/m; (b) C_1 = 0 to 300 kN s/m and (c) C_3 = 0 to 300 kN s/m.

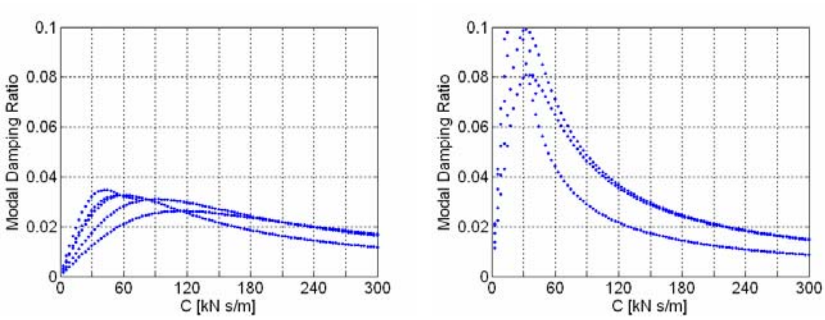

Figure 8. Modal damping ratio of first 5 modes for the case (a) C_1 = 0 to 300 kN s/m and (b) C_3 = 0 to 300 kN s/m.

The latter observation is confirmed by Fig. 8, where modal damping ratios are plotted for the first five modes only. In this case, the maximum available damping ratio is significantly larger if the damping coefficient is suitably chosen. The joint analysis of Figs. 7 and 8 shows that dampers close to the deck end (configuration C_1) are relatively more efficient in damping higher modes than dampers located away from such a position (configuration C_3). On the other hand, moving dampers toward the midspan allows to increase the efficiency in damping the lower modes.

5.5 Optimal arrangements

The above reported results suggest to use simultaneously the ground-deck lumped dampers and the dissipative handrail. In fact, while the first system is efficient on the first modes, the second one is helpful in damping the higher modes. The effectiveness in using both the systems is shown in Figs. 9(a) and 9(b), where the first 20 modal damping ratios are plotted corresponding to different point damper coefficients (3rd hanger) combined with a given value of C_d on the handrail.

Whenever the first few modes are characterized by a low frequency and one is not afraid of vandalism jokes, the additional damping of first modes can be avoided and the dissipative handrail can be sufficient to solve the problem of pedestrian induced vertical vibrations.

Even if increasing the damping coefficient of the distributed handrail dampers does not reduce the system performances, the evaluation of the optimal damping value is not usefulness. In fact, increasing the damping coefficient over a certain threshold (C_d = 30 kN s/m for the presented case study) does not increase the global performance, being the latter limited by the lumped damper effectiveness on the lower modes.

A possible way to improve the global performances is to increase the deck bending stiffness together with the handrail dampers coefficient. This possibility is shown in Fig. 9(c), corresponding to a deck girder profile HE160A instead of HE100A. An alternative solution is to use a spring-damper handrail, to increases the deck bending stiffness.

6. Numerical Reliability

The numerical reliability of the described procedure has been confirmed by time domain analyses carried out on FEM models of both the simply supported beam and the suspension footbridge. The direct integration of the whole Lagrangian equation of motion system has been used. The modal damping ratios have been evaluated by harmonically forcing the structure at a given frequency, so exciting the desired mode only, and by measuring the logarithmic decrement after the forcing stop. Analyses have been carried out for different parameter combinations, by varying the damper arrangements, the damping coefficients and the excited mode-frequency.

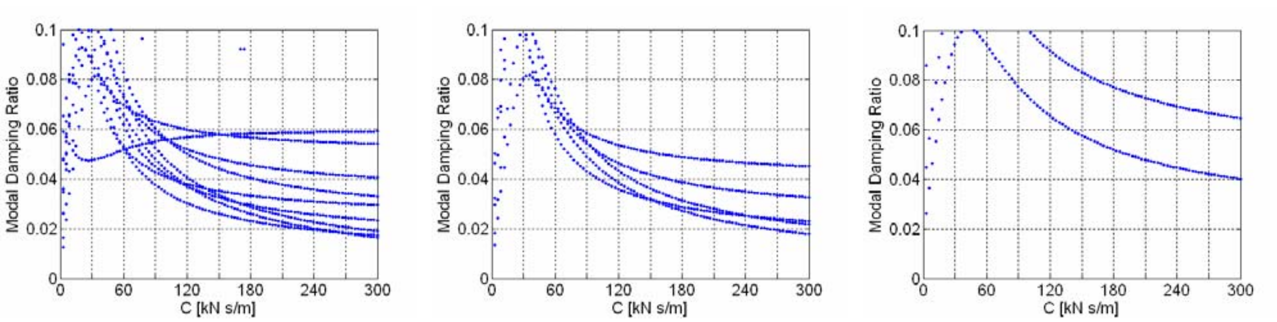

Figure 9. Modal damping ratio of first 20 modes for C_3 = 0 to 300 kN s/m and (a) C_d = 18 kN s/m, (b) C_d = 30 kN s/m and (c) C_d = 240 kN s/m with a stiffened deck girder.

7. Conclusions

Flexible pedestrian footbridges, as for instance the cable stayed and the suspension ones, are characterized by the presence of different vibration within the pedestrian excitation frequency range. The most common human induced excitations, that is the most problematic in terms of serviceability comfort, are vertical vibrations within the frequency range 1.5 to 3 Hz. In addition, pedestrian footbridges are characterized by a low inherent damping, the modal damping ratio being usually 1% to 2% as order of magnitude. Hence, to solve the comfort problem is often necessary to add external damping to the structure.

In this paper, two different damping strategies have been studied. The first one consists in placing point dampers between the ground and the bridge deck, close to the bridge ends. The second one in based on the idea of using non-structural elements to add damping to the structure; in the present case, a dissipative handrail has been hypothesized.

The damping system performances have been evaluated in terms of modal damping ratio within the frequency range interested by common vertical excitations. Point dampers create a non-diagonal modal damping matrix; since this matrix in quite sparse, the system is clearly non-classically damped and the eigenproblem solution has to be searched in the complex modal space. Furthermore, the damping optimization problem arises: in fact, too large damping coefficients behave as a restrain for the structure, without dissipating energy. In the case of distributed dampers, as the dissipative handrail is, the modal damping matrix is approximately diagonal and a "classical damping" can be hypothesized. The performances can be evaluated by the well known procedures. The modal damping increases as the damping coefficients increase.

The performed analyses showed that the point dampers exhibit good performances, if they are preliminary optimized, but only on the first few modes, up to the 5th one, in the presented case study. On the other hand, the dissipative handrail offers good performances on higher modes but is not able to damp the first ones, unless a significant over-dimensioning is adopted, so reducing the benefit-cost ratio. Subsequently, the best solution has been recognized in the joint use of the two considered systems: point dampers at the bridge ends and dissipative handrail. The procedure to analytically evaluate the optimal parameters for both the systems has been outlined and an application to a suspension footbridge has been presented.

References

[1] KOVACS I., "Zur frage de seilschwinggungen und der seil-dampfung". *Die Bautechnik* 10, 1982, pp .325-332.

[2] PACHECO M., FUJINO Y., SULEKH A., "Estimation curve for modal damping in stay cables with viscous damper". *Journal of Structural Engineering* 119:6, 1993, pp. 1961-1979.

[3] COSENTINO, N., MAJOWIECKI, M. "Analysis and mitigation of the wind induced response of large span suspended roofs: the case of the new Braga Stadium". *Proc. of 8th National Conference on Wind Engineering* Reggio Calabria, Italy, 2004.

[4] CAUGHEY, T. K., O'KELLY, M. E. J. "Classical normal modes in damped linear dynamic systems". *J. Applied Mechanics ASME*, 32, 1965, pp. 583-588.